Revue du Contrôle de la Comptabilité et de l'Audit
ISSN: 2550-469X
Numéro 6 : Septembre 2018

NON-LINEAR MODELING OF THE MOROCCAN STOCK MARKET

MODELISATION NON LINEAIRE DU MARCHE BOURSIER MAROCAIN

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ABSTRACT

Since the sixties, debates have been born on the models which determine the evolution of the stock prices. The purpose of this article is to determine how the successive variations of the MASI index are independent of each other or, in other words, whether the latter follow or not a random walk. As a result, test the hypothesis of the efficiency of the Moroccan stock market.

In order to be more precise, the choice was made for the smooth transition autoregressive (STAR) models applied to the MASI index for the period between 01/01/2004 and 31/12/2017, with data daily. It follows that they allow us to better model the nonlinear evolution of series of courses with changing regimes. This entails, therefore, the consideration of the transition, which can be slow compared to the series' adjustment dynamics, which characterizes the majority of stock market trends.

According to the results, we note that the Moroccan market does not effectively retain the information issued, which implies its inefficiency.

Keywords: Modeling, Adjustment, Nonlinear, STAR, Efficiency

RESUME

Depuis les années soixante, des débats sont nés sur les modèles qui déterminent l'évolution des cours boursiers. L'objectif de cet article est de déterminer à quel point les variations successives de l'indice MASI sont indépendantes les unes des autres ou, en d'autres termes, si ces dernières suivent ou pas une marche aléatoire "*random walk*". De ce fait, tester l'hypothèse de l'efficience du marché boursier marocain.

Pour mieux préciser, le choix s'est établi pour les modèles autorégressifs à transition lisse (STAR) appliqués à l'indice MASI pour période qui s'étale entre le 01/01/2004 et le 31/12/2017, avec des données quotidiennes. Il en découle qu'ils nous permettent de mieux modéliser l'évolution non linéaire des séries des cours ayant des régimes changeants. Ce qui entraine en conséquence, la prise en compte de la transition qui peut être lente par rapport à la dynamique d'ajustement de la série ce qui caractérise la majorité des tendances boursières.

D'après les résultats, on relève que le marché marocain ne retient pas de manière efficace les informations émises, ce qui implique son inefficience.

Mots clés : Modéliser, Ajustement, Non linéaire, STAR, Efficience

INTRODUCTION

Over the last decade, the transparency of the financial markets has become one of the major concerns that have characterized the world scene. Theorists and practitioners have been interested in this subject by studying information efficiency.

This theory is the cornerstone of modern finance, thanks mainly to its considerable contribution to the study and analysis of the transparency of financial markets. In addition, it is able to take into account all the information emitted.

The issue of information efficiency has been discussed in several previous works, our contribution in this article is concretized by, not only, evaluate to what extent can we say that the index studied is efficient? But, also, present a nonlinear modeling?

Through our problematic, we therefore pose the following hypotheses:

H1: The efficiency in the weak sense is verified for the series studied.

H2: The non-linearity of stock market dynamics.

This article will be devoted to the application of the STAR modeling to the MASI index (Moroccan All Share Index), the overall indicator of the Casablanca Stock Exchange. It should be noted, indeed, that our study period is between 01/01/2004 and 31/12/2017, with daily data, a period that is crucial in the economic and financial world having most significant events.

For this purpose, we will model the series of the MASI index through nonlinear models, more precisely, the STAR (smooth transition autoregressive) models.

This choice turns out to be the most adapted to stock market fluctuations in the presence of asymmetric phenomena and impacted by the diversified transaction costs, especially in the context of the expectations of the stock market players. These elements can be explanatory; indeed, they make it possible to justify the recourse to the smooth transition between the regimes of the stock market dynamics.

More concretely, the estimation of the model is carried out by the non-linear least squares method, but before that, we propose three steps to realize the specification of the STAR models, we must first specify the autoregressive process (AR (P)). Then, we establish the tests to validate the nonlinearity of the series studied, and finally, we specify the choice between the exponential or logarithmic models (Terasvirta, 1994). Thus the first axis of this article will study the specification of the STAR models, as for the second, it will be devoted to the estimation and validation of these models.

1- Specification of STAR models

There are multiple empirical studies on nonlinear modeling aimed at the analysis of informational efficiency. According to (Shiller 1981), stock prices are not explained by fundamentals. This conclusion was confirmed by the study of (Boussedra 2017) applied to the Moroccan currency market. For its part, (Sarantis 2001) used the STAR models to analyze the dynamics of the G7 annualized series of annualized market returns.

The STAR models are part of threshold transition models that have the capacity to record the asymmetric market dynamics in the presence of two regimes. In other words, it is a category of model that takes into account the presence of asymmetrical irregularities that dominate the stock market series.

STAR model is formulated by the following equation:

$$Y_t = \left(\alpha_{10} + \alpha_{11} y_{t-1} + \cdots + \alpha_{1p} y_{t-p}\right) + \left(\alpha_{20} + \alpha_{21} y_{t-1} + \cdots + \alpha_{2p} y_{t-p}\right) \times F(s_t, \gamma, c) + \varepsilon_t$$

The specification of the model is the fundamental step that precedes its estimation. It itself divided into three stages. First, we determine the autoregressive process (AR (P)), namely, the number of delays to be considered. Then, the linearity tests are carried out to justify the use of the STAR models. Finally, tests are conducted to orientate either towards an exponential (ESTAR) or logarithmic (LSTAR) process.

1.1- Determination of the autoregressive process (AR)

The modeling of the STAR processes is based on the determination of the number of delays resulting from the autoregressive process (AR (P)) which must be formally stationary. According to Terasvirta, we retain the delay "P" of the series which makes it possible to minimize the criteria (Akaike and Schwarz) (Terasvirta, 1994)). This requires us to check the stationarity of the series studied.

To identify the type of processes, the unit root test must be used. As a result, the ADF test is presented in three models using ordinary least squares (OLS) estimation under the hypothesis $\emptyset_1 < 1$:

$\Delta x_t = \rho x_{t-1} - \sum_{j=2}^{p} \emptyset_1 \Delta x_{t-j+1} + \varepsilon_t$: Model [1].

$\Delta x_t = \rho x_{t-1} - \sum_{j=2}^{p} \emptyset_1 \Delta x_{t-j+1} + c + \varepsilon_t$: Model [2].

$\Delta x_t = \rho x_{t-1} - \sum_{j=2}^{p} \emptyset_1 \Delta x_{t-j+1} + c + bt + \varepsilon_t$: Model [3].

Table N ° 1: Synthesis of the ADF tests of the MASI index:

Hypothesis H0 : MASI has a unit root		
Number of delays = 4 ; Critère : Akaike		
Test DFA	t Statistic	Critical probability
Model [1]	0.843579	0.8927
Model [2]	-0.543859	0.8801
Model [3]	-1.503279	0.8287

Source: Established by the author

From the table, we see that the set of critical probabilities exceed the threshold of 0.05. We therefore retain the hypothesis H0 which states that the MASI series is not stationary and has a unit root.

As a result, the filtered series is established for which the ADF test would be performed, and then its correlogram is presented to select the number of delays in order to eliminate the residual autocorrelation and to make the error process a white noise.

Figure N ° 1: ADF test of the filtered series of the MASI index

Null Hypothesis: DMASI has a unit root
Exogenous: None
Lag Length: 0 (Fixed)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-40.65534	0.0000
Test critical values:	1% level	-2.565735	
	5% level	-1.940930	
	10% level	-1.616628	

*MacKinnon (1996) one-sided p-values.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DMASI(-1)	-0.711861	0.017510	-40.65534	0.0000

R-squared	0.355923	Mean dependent var	0.007216
Adjusted R-squared	0.355923	S.D. dependent var	91.77289
S.E. of regression	73.65178	Akaike info criterion	11.43691
Sum squared resid	16224932	Schwarz criterion	11.43891
Log likelihood	-17108.61	Durbin-Watson stat	1.966266

Source: Established by the author

According to this model, we report that the variable is significant at the thresholds of:

- 1% : t-statistic |-40,65534| > |-2,565714| for the variable (DMASI(-1)).
- 5% : t-statistic |-40,65534| > |-1,940927| for the variable (DMASI(-1)).
- 10% : t-statistic |-40,65534| > |-1,616630| for the variable (DMASI(-1)).

From these results, we can confirm the stationarity of the filtered MASI series.

The following figure presents the correlogram of the filtered series of the MASI index:

Figure N ° 2: Presentation of the correlogram of the filtered series of the MASI index

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.288	0.288	247.94	0.000
		2	0.028	-0.060	250.32	0.000
		3	-0.061	-0.057	261.30	0.000
		4	-0.024	0.013	262.95	0.000
		5	-0.022	-0.021	264.35	0.000
		6	-0.037	-0.032	268.35	0.000
		7	-0.030	-0.012	271.08	0.000
		8	0.002	0.013	271.09	0.000
		9	-0.002	-0.012	271.10	0.000
		10	0.013	0.015	271.63	0.000
		11	0.029	0.024	274.22	0.000
		12	0.005	-0.015	274.28	0.000
		13	0.005	0.009	274.36	0.000
		14	0.012	0.013	274.79	0.000
		15	0.041	0.036	279.78	0.000

Source: Established by the author

According to the partial correlogram, the representation of this model can be carried out through the following model: AR (3). Since the first three delays of the filtered series are greater than 0.05.

This existence of statistical delays that differ from 0 has a major consequence such is the presence of a form of dependence between the data of the series of the MASI index. That is, the assumption that there is independence and informational efficiency for the series of the index under study must be rejected. To specify the nature of this dependence, one will apply the linearity tests before estimating the appropriate model.

1.2- Linearity tests for STAR modeling

These different tests have been widely developed by (Van Dijk, Franses and Terasvirta, 2002) and (Jawadi & Koubaa, 2006).

1.2.1- The standard linearity tests

The purpose of these tests is to detect non-linearity or linearity in establishing STAR models. The hypothesis H0 according to these tests is when the autoregressive parameters α_1 et α_2 of the regimes of the STAR model are the same. Another formulation is valid, namely: H0: $\gamma = 0$.

As we have just shown, there are several formulations of the null hypothesis H0. As a result, this may adversely affect the results of these tests.

Another method is suggested, namely to present the approximate formula of Taylor for the transition function $F(s_t, \gamma, c)$. At this level, Lagrange multipliers (LM) are used for this test.

The statistic of this multiplier (LM) is presented according to an asymptotic distribution with respect to the null hypothesis (H0) from the Chi-square law (Saikkonen & Luukkonen,1988).

Revue du Contrôle de la Comptabilité et de l'Audit
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In order to present the linearity tests with respect to the LSTAR logistic processes, it is suggested to use the first-order Taylor structural formula which represents the LSTAR model on the parameter $\gamma = 0$.

Indeed, one must test the linearity for all the realizable values of the parameter d which represents the delay, if one supposes the endogeneity of the variable of transition noted y_{t-d} in this case, one formulates the following equation:

$$y_t = \delta_0' z_t + \sum_{i=1}^{p} \sum_{j=1}^{p} \delta_{1ij} y_{t-i}\, y_{t-j} + \eta_t$$

According to the LM1 test, if $\delta_1 = 0$ at the level of the equation, we assume the linearity assumption. On the other hand, we know that δ_1 is dependent only on $\alpha_{2,j}$ (j varies from 1 to p). So it does not depend on $\alpha_{2,0}$.

This means that the test loses its significance when $\alpha_{2,0} = 0$. In other words, the LM1 test does not make it possible to check the non-linearity which limits its use.

To cope with this insufficiency, we suggest the change of the transition function $F(s_t, \gamma, c)$ by Taylor's structural formula of the order of 3 with respect to $\gamma = 0$. The null hypothesis H0 is verified, in this case, by the following condition: H0 : $\delta_1 = \delta_2 = \delta_3 = 0$. This allows us the formulation of the regression, called auxiliary, following:

$$y_t = \sum_{i=0}^{p} \delta_{0i}\, y_{t-i} + \sum_{i=1}^{p} \sum_{j=1}^{p} \delta_{1ij} y_{t-i}\, y_{t-j} + \sum_{i=1}^{p} \sum_{j=1}^{p} \delta_{ij} y_{t-i}\, y_{t-i}^2 + \sum_{i=1}^{p} \sum_{j=1}^{p} \delta_{1ij} y_{t-i}\, y_{t-i}^3 + \eta_t$$

The Lagrangian test noted LM3 is made from the following steps:

Step 1: The residues are formulated as follows:

$$SCR_0 = \sum_{t=1}^{T} \widehat{\varepsilon_t^2}$$

They are from Ordinary Least Squares (OLS) regression of y_t over z_t.

Step 2: We use the regression noted "auxiliary" for the estimation of y_t on z_t with t = i (∀i = 1,2,3). This allows us to formulate η_t as follows:

$$SCR_3 = \sum_{t=1}^{T} \widehat{\eta_t^2}$$

Step 3: We formalize LM3 which represents the Lagrangian test:

$$LM_3 = \frac{t(SCR_0 - SCR_3)}{SCR_0} \rightarrow \chi^2(3p)$$

For the linearity tests for the ESTAR exponential model, the same deficiencies identified by the LSTAR logistic models are retained. As a result, we propose to use Taylor's structural formula order 1 according to the following equation:

$$y_t = \delta_0' z_t + \delta_1' z_t s_t + \delta_2' z_t s_t^2 + \eta_t,$$

As mentioned above, the linearity is validated when the parameters δ_1 and δ_2 are equal to the level of the formula above. This allows us to formulate the Lagrange multiplier order 2 noted LM2:

$$LM_2 = \frac{t(SCR_0 - SCR_2)}{SCR_0}$$

It is specified that the sum of the squares of the residues (SCR), represented in the formula, verifies that LM2 is an asymptotic distribution with respect to a Chi-square law.

The same shortcomings of this test and the use of Taylor's structural formula are noted to account for inflection points related to the exponential process function. Hereinafter, we present the formula of the so-called "auxiliary" regression (Escribano & Jorda, 1999):

$$y_t = \delta_0' z_t + \delta_1' z_t s_t + \delta_2' z_t s_t^2 + \delta_3' z_t s_t^3 + \delta_4' z_t s_t^4 + \eta_t$$

As a result, the process is considered linear if the null hypothesis (H0) satisfies:

H0 : $\delta_1 = \delta_2 = \delta_3 = \delta_3 = 0$.

We note the Lagrange multiplier LM4 must be in the form of an asymptotic distribution of the Chi-square law. The following are the standard linearity tests in the following table:

Table n ° 2: Results of standard linearity tests (p_values) of the MASI index,

P_value / LM	d=1	d=2	d=3
LM1	0.162	0.125	0.007
LM2	0.117	0.044	0.001
LM3	0.153	0.127	0.000
LMe3	0.251	0.181	0.000
LM4	0.345	0.206	0.008

Source: Established by the author

The table above represents the results of linearity tests compared to the Moroccan stock market, which allows us to report the dynamics of this series. The results of the various tests of the Lagrange multiplier (LM1, LM2, LM3, LMe3, LM4) demonstrate the non-linearity of the process of our studied series. This allows us to validate the adaptation of the STAR models for the MASI index.

1.2.2- Conditional heteroscedasticity analysis related to the ARCH effect

The analysis of the ARCH effect and the phenomenon of conditional heteroscedasticity is essential in the study of the theory of information efficiency.

This leads to a link between the presence of the ARCH effect and the random walk hypothesis, which can be explained by the relationship between the independence of the conditional variance and the dynamics of the stock market.

Indeed, we can present the ARCH effect as a temporal dependence relation of the conditional variance of the studied series. The consequence of the validation of the presence of the ARCH effect of the series therefore leads to the possibility of predicting the future risk represented by the variance of the series studied through the variances calculated on the basis of past prices, which refutes the hypothesis of a random walk, that is, informational efficiency.

The stochastic process is auto-correlated if the dynamics of the series depend on time. In our study of the Moroccan index, we check if the stock market series is time dependent. In case we confirm this result, we conclude for the presence of the ARCH effect. More concretely, the ARCH effect test is determined from the autocorrelation of the residue squares (ε_t^2).

The simple model is formulated as follows in case of presence of the ARCH effect in the studied series:

$$\varepsilon_t = z_t\sqrt{h_t}$$

$$h_t = \alpha_0 + \alpha_1\ \varepsilon_{t-1}^2$$

This allows us to formulate the GARCH process (1,1) as follows:

$$\varepsilon_t = z_t\sqrt{h_t}$$

$$h_t = \alpha_0 + \alpha_1\ \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$$

In this case, z_t is set as a white noise in the weak form with the following conditions: $E(z_t) = 0$ and $Var(z_t) = \sigma_\tau^2$.

Also, we specify that h_t constitutes the conditional variance resulting from the MASI series dependent on the delayed errors ε_{t-1} according to the ARCH (1) model and of ε_{t-1}, and it also depends on h_{t-1} in the GARCH model (1,1) and we write ε_t as the error residuals from the MASI series, with $\propto_0 > 0, \propto_1 \geq 0$ and $\beta_1 \geq 0$.

There are two types of tests for determining the ARCH effect:

- Tests based on the formulas of the usual Q_{stat} (Box Pierce, Ljung Box, ...). Called autocorrelation tests on squares ε_t.
- LM (Lagrange Multiplicateuur) tests detecting the absence of autocorrelation linked to residues ε_t.

Therefore, the residue square process is formulated as follows:

$$\hat{\varepsilon}_t^2 = \beta_0 + \beta_1 \hat{\varepsilon}_{t-1}^2 + \cdots + \hat{\varepsilon}_{t-k}^2 + \mu_t$$

$\hat{\varepsilon}_{t-j}^2$ is the square of the estimated residues from the MASI series. The ARCH effect test is based on the verification of the following assumptions:

$$H_0: \beta_1 = \beta_1 = \cdots = \beta_k = 0$$
$$H_0: \beta_i \neq 0 \ \forall i \in [1, k]$$

Indeed, we specify that the statistical value of the Lagrangian multiplier follows a Chi-square law under the null hypothesis H_0 when N tends to infinity $(N \to \infty)$.

If the parameters $\beta_i \ \forall i \in [1, k]$ are different from 0 in the process of the squares of the estimated residuals, in other words, H_0 is valid, the ARCH effect is considered absent in this case since the residues are not self-correlated. On the other hand, if H_0 is not valid, we admit that the ARCH effect is considered present, which naturally means that there is dependence of the series with respect to time.

The results of these tests applied on the filtered series of the MASI index are presented as follows:

Table N ° 3: The ARCH effect in the filtered series of the MASI index

Series	Order q	Qstat	ARCH effect
Total period			
DMASI	12	85 (au seuil de 1%)	YES

Source: Established by the author

It is specified that the autocorrelation order is represented by q which signifies the number of delays in the formula of the conditional variance. The statistical value of the Ljung-Box test is represented by Qstat. This result validates the presence of the ARCH effect in the MASI series, which means that the Moroccan stock market process depends on time.

1.3- Precision of the transition function

Since the linearity tests confirm the adequacy of the STAR models. In what follows, we will establish tests that allow specifying the most suitable process among the logistic or exponential models. Indeed, these tests make it possible to compare the auxiliary regressions at the level of the linearity compared to the models LSTAR or ESTAR. More explicitly, the following assumptions are made to fit the established tests:

$$\mathrm{H01}: \delta_1 = 0/\delta_3 = \delta_2 = 0$$

$$\mathrm{H02}: \delta_2 = 0/\delta_3 = 0$$

$$\mathrm{H03}: \delta_3 = 0$$

In other words, this consists of testing the parameters $\delta_i (\forall i = 1,2,3)$ at the level of the formula below:

$$y_t = \delta_0' z_t + \sum_{i=1}^{p} \sum_{j=1}^{p} \delta_{1ij} y_{t-i}\, y_{t-j} + \eta_t$$

The basic principle of this test is to check the following situations:

- It is an LSTAR model if we reject the two hypotheses H01 and H02 more strongly than the hypothesis H03.
- This is an ESTAR model if we reject hypothesis H02 more strongly than both hypotheses H03 and H01.

To deepen our analysis, we refer to the works of Escribano and Jorda (1999) who proposed the use of the Lagrange multiplier test order 4 (LM4) to deduce the choice between the two processes LSTAR and ESTAR through the characteristics of parameters $\delta_i (\forall i = 1,2,3,4)$ relating to the following formula:

$$y_t = \delta_0' z_t + \delta_1' z_t s_t + \delta_2' z_t s_t^2 + \delta_3' z_t s_t^3 + \delta_4' z_t s_t^4 + \eta_t$$

The hypotheses of this test are formulated as follows:

$$\mathrm{H0E}: \delta_2 = \delta_4 = 0$$

$$\mathrm{H0L}: \delta_2 = \delta_3 = 0$$

As a result, we retain the LSTAR logistic model in the case where the quadratic terms of orders 1 and 2 are null, but if they are present, we take the ESTAR exponential model.

In the following, the results of this test are presented according to this table:

Table 4: Transition Function Selection Test Results (d = 3)

Standard tests				
Teräsvirta (1994)			Escribano et Jordã (1999)	
H03	H02	H01	H0L	H0E
0.00168	2.01*10-8	0.0019	0.00061	1.45*10-5
Robust tests with heteroscedasticity				
Teräsvirta (1994)			Escribano et Jordã (1999)	
H03	H02	H01	H0L	H0E
0.1	0.0313	0.191	0.0665	0.0632
Robust tests with outliers				
Teräsvirta (1994)			Escribano et Jordã (1999)	
H03	H02	H01	H0L	H0E
0.00233	0.317	0.652	0.00178	0.202

Source: Established by the author

Based on the results of standard tests and robust outlier tests, ESTAR models are found to be the most appropriate for reproducing the dynamics of the MASI index series for the chosen transition y_{t-3}.

Thus, we opt for the exponential transition function that is noted ESTAR for our model relating to the series of the MASI stock index. These results confirm those of (Dumas, 1992) and (Anderson, 1997). Depending on their work, this can be explained by the asymmetry effect linked to the stock market series and the presence of transaction costs.

These results confirm other more recent studies that have concluded for the choice of exponential models with respect to logistic or even linear models in order to better relate the dynamics of the stock index processes. The work of Jawadi and Koubaa (2006) [dealing with the dynamics of stock market returns] is cited here.

2- The estimation and validation of STAR models for the MASI index

After having specified the model and checked the non-linearity tests of the Moroccan stock market, we establish, in a first paragraph, the STAR model for the series of the Moroccan stock market index, as for the second, it will be devoted to its validation.

2.1- Estimation of the STAR models for the MASI index

According to previous tests, we have validated the non-linearity of the series studied, therefore, our estimation method will be carried out by the non-linear least squares method. Our method is inspired by the works of (Van Dijk, al, 2002). This method is carried out through the following steps: firstly, the concentration of the function of the sums of squares is performed, then the initial values of the parameters of the model are chosen, and finally, the transition speed is estimated determined by the parameter γ.

The value of this parameter is very sensitive at the estimation level of STAR models. According to (Terasvirta, 1994), if γ takes a high value, it means that the convergence of the model is blocked. More precisely, it is the optimization algorithm that does not converge. To cope, the author suggests a procedure that depends on the choice of model. In the case of the exponential model, the smoothing parameter must be standardized by its variance. As for the logistic model, the standard deviation is used.

Indeed, we propose the following formula for the ESTAR model in order to estimate the Moroccan stock market series:

$$F(s_t, \gamma, c) = 1 - \exp\left\{\frac{\gamma}{\widehat{\sigma^2_{s_t}}}(s_t - c)^2\right\}, \gamma > 0$$

Through this formalization, the initial value of γ would be close to unity (1), which is also a risk for the convergence of the model algorithm. To cope with this, values around the unit were tested at the ESTAR exponential process estimation. The change of values is reproduced until the stabilization of the various parameters of the estimated process.

At the empirical level, the following is an estimate of the ESTAR model (3.3) of the MASI index series in the following table:

Table 5: ESTAR model estimation results for the MASI index

Parameters	Estimated values	Estimated standard deviations robust to heteroscedasticity.
α_{10}	0.00284	(0.00015*)
α_{11}	0.3127	(0.0268*)
α_{12}	0.155	(0.0366*)
α_{13}	0.0945	(0.0206*)
α_{20}	$2.845 * 10^{-5}$	(0.00086*)
α_{21}	0.47	(0.0788*)
α_{22}	0.2566	(0.0849**)
α_{23}	0.1141	(0.0885**)
γ	6.5	(49*)
c	0.0076	(0.00057*)
SCR	0.00485	
$\gamma*\sigma^2$	0.062	
J-B	1460 p_value (0.00)	
ARCH(1)	7.645 p_value (0.01)	
ARCH(4)	49.52 p_value (0.00)	
(*) and (**) indicate the significance of the parameters estimated respectively at the 5% and 10% thresholds.		

Source: Established by the author

The relevance of the choice estimation by the ESTAR model is reflected by the results of the table. This confirms the ability of this process to reflect the dynamics of the stock market series, which is perfectly suited to the results of the studies conducted by (Jawadi & Koubaa, 2006) and (Sarantis, 2001) at the STAR model level.

More concretely, we note that the significativities of the parameters γ and c. The impact of this result is to validate the presence of two regimes and the so-called smooth transition as hypothesis of our model. We can therefore see that the ESTAR model allows to better relate the process of the Moroccan series with two regimes with a threshold. It is also concluded that the value of parameter c equivalent to 0.0076 which is significant and positive representing the profitability of 16/09/2009. This can be interpreted through the existence of 1326 observations, representing 36.44% below the threshold and 2314 above.

At the economic level, one can analyze the change of regime through the various national or international events having directly or indirectly impacted the evolution of the Moroccan stock market series (Jawadi & Koubaa, 2006)).

In general, the strong activity and dynamism of the Casablanca Stock Exchange can be described during the period from January 2004 to mid-March 2008.

It is recalled at this level that the MASI index is the indicator that includes all the securities of the Moroccan market, which allows it to constitute an important economic indicator. This index has achieved a significant change which is illustrated by a rate of 71.6% in 2006. This regime has generated concerns that are materialized by a bearish trend in the following period.

At the level of Moroccan internal politics, this period stood out by the municipal elections. In addition the sovereign revived the commitment of Morocco to the defense of the environment and sustainable development as a regional and African leader (...). On the other hand, the change of the regime was illustrated by the impact of the crisis weighed down by the fall in foreign investment.

As for the international scene, it was mainly characterized by the upward trend in oil prices accompanied by the decline of the US dollar against the euro and the Japanese yen.

At the econometric level, some authors admit the difficulty of interpreting the parameters of STAR models. Therefore, it would be wise to proceed by analyzing the parameters of the so-called external regimes. Their identification is established from the separation between the central regime and extreme regimes. The dynamics of the central regime are thus presented from a stationary autoregressive model (AR (3)) formulated as follows:

$$y_t = 0.00284 + 0.3127y_{t-1} + 0.155y_{t-2} + 0.0945y_{t-3}$$

This regime is marked by the local stability of its dynamics. This is linked to intense variations in the returns of the stock market which can be explained by the lack of active and continuous arbitrage. The deviation of prices from the fundamental value intensively can be reproduced by this regime. Indeed, the dispersion and misalignment of prices in relation to the equilibrium because of the discontinuity of the arbitrations essentially related to the difference between the expected returns and transaction costs applied and the actions of the chartists.

Concerning extreme regimes, their dynamics are different whatever they are represented by an autoregressive process (AR (3)). These are much less stable than the central regime, insofar as these regimes are distinguished by returns to balance more strongly compared to the steps initiated by fundamentalists. This helps identify the alignment of prices to equilibrium values in a more persistent way.

To better confirm our results and refine our analysis, we go on, in the following, validity tests ESTAR models.

2.2- Validation test of ESTAR estimated models

In order to validate the model estimated for the MASI stock index, we retain three specification tests which are inspired by the work done by (Eitrheim and Teräsvirta, 1996), namely the tests of absence of residual autocorrelation, the tests of no -linearity remaining and the tests of constancy of the parameters.

2.2.1- Residual autocorrelation tests

The origin of this test goes back to the work of (Godfrey, 1979), these tests are generated from the autocorrelation performed on linear models. They make it possible to test the null hypothesis (H0) which supposes that the residual autocorrelation is non-existent compared to the hypothesis of the presence of the dependence of the residues resulting from the estimation of the STAR model. We estimate the Lagrange multiplier (LM) statistic, which is interpreted by a TR ², where R ² represents the coefficient of determination of the regression of ε_t on:

$$\nabla F(z_t, \hat{\beta}) = \frac{\partial F(z_t, \beta)}{\partial \beta},$$

$$\text{with } \beta = (\alpha_2, \gamma, c)' \text{and the q delayed residues } \widehat{\varepsilon_{t-1}}, \widehat{\varepsilon_{t-2}}, \dots, \widehat{\varepsilon_{t-q}}.$$

We note the Lagrangian LM_{SI} as being the statistic of the residual autocorrelation test, we suppose at this level that it is an asymptotic distribution with respect to a Chi-square law having q degree of freedom.

2.2.2- Non-linearity tests developed

It was assumed when estimating the ESTAR model of the MASI index series that it is a dual-mode dynamic. However, this could be biased by the existence of multiple schemes. Therefore, we must ensure our basic assumption from the tests proposed in the work of (Eitrheim and Terasvirta, 1996). At this level, we propose to validate by a test whose null hypothesis supposes the existence of two regimes compared to the hypothesis which stipulates 3 regimes.

The principle of this approach is to realize by an additional transitional function formulated: $F_2(s_t, \gamma_2, c)$. It is a tester of the nonlinearity and its degree of assimilation by the model ESTAR with double regimes.

It should also be noted that our formulation may raise the problem of nuisance parameters. The solution in these situations is to formulate Taylor's development of order 3 by replacing the transition function.

Therefore, the following formula is presented:

$$y_t = \alpha_1' z_t + \alpha_2' z_t \times F_1(s_t, \gamma_1, c_1) + \delta_0' z_t + \delta_1' z_t s_t + \delta_2' z_t s_t^2 + \delta_3' z_t s_t^3 + \eta_t$$

where : δ_0' and $\delta_i', \forall i = 1,2,3$ are function of the parameters $\alpha_1', \alpha_1', \gamma_1$ et c_1

Thus, we put the null hypothesis in the following form:

H0 : $\gamma_2 = 0$ would be the equivalence of $\delta_0' = \delta_1' = \delta_2' = \delta_3' = 0$

At the statistical level, we proceed by the following steps:

- Step 1: An estimation of the series is presented using the non-linear least squares method, according to the ESTAR model.
- Step 2: we recover the series of residues to perform the following regression:

$$\nabla F(z_t, \hat{\beta}) = \frac{\partial F(z_t, \beta)}{\partial \beta}$$

We put $\beta = (\alpha_2, \gamma_1, c_1)$ and $z_t\ s_{it}, \forall i = 1,2,3$ the statistic of the test would be carried out on the basis of the coefficient of determination, noted R^2, of the regression. It is, indeed, a TR^2 formulated $LM_{AMR,3}$ considered as an asymptotic distribution according to $\chi^2(3(p+1))$.

2.2.3- Stability (or constancy) tests of estimated parameters

Our assumption, at the level of the estimation of the model for the MASI stock exchange, is based on the constancy of the parameters. Naturally, this assumption requires verification to be validated. The objective is to know if the estimated parameters are correlated with time.

(Terasvirta, 1994) and (Lundberg and Terasvirta, 1998) hypothesized that the transition function of the estimated parameters and the endogenous variable is similar. In this case, the equation of the transition function is a time variable noted: $s_t = t$ and the transition function would be of type: $F_2(t, \gamma_{pnc}, c_2)$ where γ_{pnc} is considered to be the transition speed:

$$\beta = (\alpha_2(t), \gamma, c)' \text{et } \alpha_2(t) = \alpha_{20} + F(t, \lambda\alpha_2, c\alpha_2)$$

Indeed, the parameters will be considered constant if $\gamma_{pnc} = 0$. As for the other tests, there is the risk of nuisance parameters, which leads us to propose the development of Taylor of order three.

We also note the Lagrangian $LM_{c,3}$ as being the statistic of the test obtained like $LM_{AMR,3}$ with $s_t = t$.

The following table presents the results of the application of these tests at the level of the MASI index series.

Table 6: Residual autocorrelation tests

Residual autocorrelation test (p_value of LMSI tests)	
Ordre d'autocorrélation	MASI
q = 1	0.37
q = 2	0.65
q = 4	0.25
q = 8	0.32
q = 12	0.14
Constancy test of all parameters (p_value $LMc, i, \forall\, i = 1, 2, 3$).	
$LMc, 1$	0.27
$LMc, 2$	0.36
$LMc, 3$	0.53
Residual non − linearity test (p_value de LMAMR)	
$d' = 1$	0.05
$d' = 2,$	0.18
$d' = 4$	0.35
$d' = 8$	0.98
$d' = 12$	0.12

Source: Established by the author

Like the results obtained, we note the rejection of the hypotheses related to normality and homoscedasticity of the Moroccan stock market series from the Jarque-Bera and Engle (1982) tests. In addition, it is asserted, through the tests, that the residues are heteroscedastic Gaussian. This leads us to note the existence of the ARCH effect for process residues.

It can also be stated that the estimated parameters are unbiased, based on the non-correlation tests carried out on the residues of the stock exchange studied, which proves that they are not correlated. In addition, it should be mentioned that the estimated parameters of the model do not change with respect to time, this is affirmed by the tests of constancy of the parameters.

Concerning the critical values (p values) resulting from the tests of nonlinearity remaining exceed the 5% concerning all the values of the delays except for the delay d'= 1. Indeed, the ESTAR model with dual regime fully relates the non-linearity.

All the results converge towards the relevance of the choice of the STAR modeling and more precisely the ESTAR model (3,3) chosen to relate the nonlinearity and the presence of a double regime with smooth transition for the Moroccan stock market dynamics.

Conclusion

The analysis of stock market dynamics in general and specifically that of the Moroccan stock market, the subject of this article, affected by phenomena related to transaction costs, the heterogeneity of expectations of market operators and asymmetry informational sources of information inefficiency, which therefore leads to the rejection of the weak form of informational efficiency because it does not show that it is a random walk that allows stock prices to instantly adjust linearly and symmetrically. Quite naturally, this means that efficiency is rejected in its three forms: weak, semi-strong and strong.

The analysis of the causes of this inefficiency informs the situation of the structural components of this market. If we study the impact of transaction costs, we deduce that they have effects limiting arbitrage and the adjustment of stock prices by reducing the volume of transactions. They also create deviation effects, reported during the analysis of our model. This makes our choice of nonlinear models relevant because linearity leads to bias analysis.

The estimation of the Moroccan stock market by the class of STAR models, could confirm the relevance of this choice compared to linear models to better relate the stock market dynamics and its evolution. Especially in the presence of transaction costs, asymmetric phenomena and the non-linear characteristics inherent in the stock market process. Indeed, the various tests, from the specification, then the estimation and finally the validation, validated the ESTAR exponential model (3.3) for the Moroccan series MASI.

To better explain the validation of the ESTAR model for the Moroccan stock market, we can evoke the weakness of orders and transaction volumes made at the level of the Casablanca stock exchange associated with the heterogeneity of expectations and behavior of Moroccan stock market operators. . In terms of the intrinsic continuity of the estimation of the ESTAR exponential model, it provides information on the heterogeneity of operators in the Moroccan stock market, the heaviness and the respite effects characterizing the diversified transaction costs.

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